Workshop *Stochastic Geometry and its Applications*
March 28-30 2012, Rouen

François Baccelli, INRIA Paris-ENS, France

Sur les processus spatiaux de naissance et de mort du pair-à-pair

Nous étudions une classe de processus de naissance et de mort de l'espace euclidien où la dynamique est celle des réseaux pair-à-pair de l'Internet : à tout instant, le "taux de mort/départ" d'un point donné est proportionnel à l'intégrale d'une fonction positive (la fonction d'echange de ce point) par rapport au processus ponctuel à cet instant.

Nous montrons que le processus ponctuel décrivant l'état d'équilibre de ce processus de naissance et de mort est "répulsif".

Nous analysons deux régimes asymptotiques : le régime fluide et le régime avec exclusion de type boules dures.

Les résultats sont fondés sur l'analyse markovienne, sur la théorie des processus ponctuels (probabilités de Palm et intensité stochastique) et sur l'analyse dimensionnelle.

Travail en collaboration avec Fabien Mathieu (Paris) et Ilkka Norros (Helsinki).

Hermine Biermé, Université Paris Descartes et Université de Tours

A turning-band method for the simulation of anisotropic fractional Brownian fields

In medical imaging, several authors have proposed to characterize roughness of observed textures by their fractal dimensions. These studies are based on the stochastic modeling of images using the famous fractional Brownian field for which the fractal dimension is determined by its so-called Hurst parameter. However, this stochastic model does not allow to reveal texture anisotropy which may be an important characteristic for diagnostic help.

We consider anisotropic generalizations of this model, obtained and characterized by an anisotropic deformation of the fractional Brownian field spectral density. We focus on the issue of simulating realizations on a regular grid. Initially suggested by Matheron, the turning-band method is a generic method which reduces the problem of simulating a stationary isotropic Gaussian field in several dimensions to the problem of simulating several processes in one dimension. We propose an adaptation of this method in our 2D anisotropic and non stationary framework, based on fast and exact synthesis of 1D fractional Brownian motion. This is joint work with Lionel Moisan (MAP5, Université Paris Descartes, France) and Frédéric Richard (LATP, Aix-Marseille Université, France).

Bartek Błaszczyszyn, INRIA Paris/ENS, France
On comparison of clustering properties of point processes

We propose a new comparison tool for spatial homogeneity of point processes, based on the joint examination of void probabilities and factorial moment measures. We prove that determinantal and permanental processes, as well as, more generally, negatively and positively associated point processes are comparable in this sense to the Poisson point process of the same mean measure. We provide some motivating results and preview further ones, showing that the new tool is relevant in the study of macroscopic, percolative properties of point processes. This new comparison is also implied by the directionally convex (dcx) ordering of point processes, which has already been shown to be relevant to comparison of spatial homogeneity of point processes. For this latter ordering, using a notion of lattice perturbation, we provide a large monotone spectrum of comparable point processes, ranging from periodic grids to Cox processes, and encompassing Poisson point process as well. They are intended to serve as a platform for further theoretical and numerical studies of clustering, as well as simple models of random point patterns to be used in applications where neither complete regularity nor the total independence property are not realistic assumptions.

This is joint work with D. Yogeshwaran [Technion].

Christian Buchta, University of Salzburg, Austria
The convex hull of randomly chosen points

The talk starts with a classical result by Rényi and Sulanke from the early sixties : Assume that n points are chosen independently and uniformly from the interior of a convex polygon with r vertices. Then the expected number of vertices of the convex hull is of order log n as n tends to infinity. Furthermore, it is asymptotically proportional to r. Various lines of research from the past fifty years are considered, including

- refinements of the original result,
- extension to dimension three,
- extension to higher dimensions,
- variance,
- higher moments,
- exact distribution.

The emphasis is put on recent results and current developments.

Youri Davydov, Université Lille 1, France
On random symmetrizations of convex bodies

We consider sequences of Steiner (and Minkowski) symmetrizations of convex bodies in R^d. It is known that if the corresponding directions are chosen at random, these symmetrizations converge a.s. and the limit sets are balls. The aim of our talk is to discuss the rate of this convergence. In particular, we prove that in dimensions $d = 2, 3$ the Steiner and Minkowski random symmetrizations converge exponentially fast. Moreover, this result remains true when the uniform distribution of directions is replaced with a distribution from some more large class.

This is joint work with David Coupier.

David Dereudre, Université Lille 1

Continuum percolation and phase transition for multy-type Quermass Model

We present results involving continuum percolation for Markov (or Gibbs) germ-grain models. The grains are assumed circular with random radii on a compact support. The morphological interaction is the so-called Quermass interaction defined by a linear combination of the classical Minkowski functionals (area, perimeter and Euler-Poincaré characteristic). We show that the percolation occurs for any coefficient of this linear combination and for a large enough activity parameter. An application to the phase transition of the multi-type Quermass model is given. It is a joint work with David Coupier.

Catherine Gloaguen, France Telecom Orange Labs, Issy-Moulineaux

Parametric Distributions of Connection Lengths for the Efficient Analysis of Fixed Access Networks

The access network displays the important particularity that the locations of network components strongly depend on geometrical features such as road systems and a city's architecture. This talk deals with the distributions of point-to-point connection lengths that play a major role in current problems of the analysis and planning of networks. Using the mathematical framework of stochastic geometry to model both the road system and the locations of network nodes, we derive analytical formulas for distributions of typical connection lengths, see [1]. These formulas only depend on a few parameters that can be computed easily and fast avoiding time consuming reconstructions, where the road system is modelled by random tessellations of Poisson type, and the locations of network nodes by Cox processes on the edges of the tessellations.

We also introduce some new random objects of interest for network modeling : extensions to non-Euclidean (graph-theoretic) connection lengths and to road models which allow dead ends as well as non-convex cells, see [2]. Random geometric graphs are the subject of another talk presented during this workshop (by V. Schmidt).

[1] Gloaguen, C., Voss, F. and Schmidt, V. (2011) Parametric distributions of connection lengths for the efficient analysis of fixed access networks. Annals of Telecommunications 66, 103-118.

[2] Neuhaeuser, D., Hirsch, C., Gloaguen, C. and Schmidt, V. (2012) On the distribution of typical shortest-path lengths in connected random geometric graphs. Queueing Systems (in print).

Xavier Goaoc, INRIA Nancy

Raising expectations about random polytopes

A random polytope K(n) is usually defined as the convex hull of n points distributed independently and uniformly in a compact convex body K. The study of random polytopes goes back to Sylvester's "four points problem", which asked for the probability that four points chosen at random be in convex position. In this talk, I will present some old and some new results on the question of whether the expected complexity of K(n) is an increasing function of n.

Daniel Hug, University of Karlsruhe, Germany

On the Zero Cell of a parametric class of Poisson Hyperplane Tessellations

We investigate the size and shape of the zero cell (the cell containing the origin) of a parametric class of isotropic but not necessarily stationary Poisson hyperplane tessellations in n-dimensional Euclidean space. Our focus is on the volume of the zero cell and its intersections with lower-dimensional balls and subspaces. In particular, we obtain an explicit formula for the variance of the volume of the zero cell in arbitrary dimensions by using methods of integral geometry. From this formula we deduce e.g. the asymptotic behaviour of the volume of the zero cell as the dimension goes to infinity. Our formula can also be used for explicit numerical calculations, and to obtain yet another partial confirmation of the slicing conjecture for a certain class of random polytopes.

This is joint work with Julia Hörrmann.

Dominique Jeulin, Mines ParisTech

Multi-scale random sets for physical applications

Complex microstructures in materials often involve multi-scale heterogeneous textures, modelled by random sets derived from Mathematical Morphology [1]. Starting from 2D or 3D images, a complete morphological characterization by image analysis is performed, and used for the identification of a model of random structure [2]. From morphological models, simulations of realistic microstructures are introduced in a numerical solver to compute appropriate fields (electric, elastic stress or strain, . . .) and to estimate the effective properties by numerical homogenization, accounting for scale dependent statistical fluctuations of the fields [3]. Our approach is illustrated by various examples of multi-scale models : combination of random sets, Boolean random sets based on Cox point processes and various random grains (spheres, cylinders), showing a very low percolation threshold, and therefore a high conductivity or high elastic moduli for a low volume fraction of a second phase [4]. Multiscale Cox point processes are also a source of instructive models of fracture statistics, such as multicriteria and multiscale weakest link models [5].

Références

[1] D. Jeulin, Multi Scale Random Models of Complex Microstructures, in : T. Chandra, N. Wanderka, W. Reimers, and M. Ionescu (Eds), Thermec 2009 (Materials Science Forum, Vol. 638 - 642), 81-86.

[2] A. Jean, D. Jeulin, S. Forest, S. Cantournet, F. N'Guyen, A multiscale microstructure model of carbon black distribution in rubber J. Microsc. 241 (3) (2011) 243- 260.

[3] F. Willot, D. Jeulin, Elastic and Electrical Behavior of Some Random Multiscale Highly Contrasted Composites, International Journal of Multiscale Computational Engineering, Vol. 9 (3) (2011) 305-326.

[4] Morphology and effective properties of multi-scale random sets : a review, accepted for publication in Comptes-Rendus Mécanique, Académie des Sciences, Paris (2012).

[5] D. Jeulin, Multi-scale random sets : from morphology to effective properties and to fracture statistics,in : Proc. CMDS12, J. Phys. : Conf. Ser. 319 012013, (2011) ; doi :10.1088/1742-6596/319/1/012013.

Frédéric Lavancier, Nantes University
Statistical aspects of determinantal point processes

Determinantal point processes are largely unexplored in statistics, though they possess a number of appealing properties and have been studied in mathematical physics, combinatorics, and random matrix theory. In this talk we consider statistical aspects of determinantal point processes defined on $\mathbb{R}^d$, with a focus on $d = 2$. Determinantal point processes are defined by a function C satisfying certain regularity conditions and they possess the following properties :

(a) Determinantal point processes are flexible models for repulsive interaction.

(b) All orders of moments of a determinantal point process are described by certain determinants of matrices with entries given in terms of C.

(c) A one-to-one smooth transformation or an independent thinning of a determinantal point process is also a determinantal point process.

(d) A determinantal point process can easily be simulated, since it is a mixture of 'determinantal projection processes'.

(e) A determinantal point process restricted to a compact set has a density (with respect to a Poisson process) which can be expressed in closed form including the normalizing constant.

In contrast, Gibbs point processes, which constitute the usual class of models for repulsive interaction, do not in general have moments that are expressible in closed form, their density involves an intractable normalizing constant, and rather time con- suming Markov chain Monte Carlo methods are needed for simulations and approximate likelihood inference. In this work we describe how to simulate determinantal point processes in practice and investigate how to construct parametric models. Furthermore, different inferential approaches based on both moments and the likelihood are studied. The work has been carried out in collaboration with Jesper Møller and Ege Rubak from Aalborg University (Denmark).

Jean-Francois Marckert, Université Bordeaux 1
Navigation on a Poisson point process

A navigation on a set of points is a rule for choosing which point to move to in order to progress toward a specified target. In this work, we study some geometrical based navigations in $\mathbb{R}^2$. In particular, we are interested in asymptotic results, when the number of points goes to infinity, and are chosen according to a probability distribution with a bounded support. We obtain asymptotic results concerning the asymptotic geometry of the navigations paths, their asymptotic lengths, their number of stages, and their behaviour with respect to other various cost functions. This is joint work with Nicolas Bonichon.

Laurent Massoulié, Technicolor Paris
Loss Networks and Matchings in Random Environments

We consider large scale distributed server systems, featuring both memory and bandwidth resource constraints. Our aim is to identify memory management strategies which maximize overall system capacity.

We first propose the so-called proportional content placement strategy, and characterize its efficiency in a large system asymptotic regime. This result features a non-classical loss network in a random environment.

An alternative model is considered next, which measures performance via the size of matchings in suitable graphs. In this context, we first show the impact of the matching strategy by comparing greedy and optimal matchings. In particular, we show that greedy matching undergoes a phase transition, with a severe performance degradation, at critical loading. We also investigate improvements on the previous "proportional" placement strategy. Underlying these results is a characterization of density of maximum matchings in large bipartite random graphs.

This presentation is based on joint work with Marc Lelarge (INRIA/ENS), Mathieu Leconte (Technicolor/INRIA) and Bo Tan (UIUC).

Rahul Roy, Indian Statistical Institute, Delhi, India

Finite clusters in a high intensity Zwanzig percolation model

Zwanzig (1963) studied a system of non-overlapping hard needles in the continuum, where the orientation of the needles were restricted to a finite set. Here he observed that as the density of needles increased a phase transition occurred from an isotropic phase, where the rods are placed 'chaotically', to a nematic phase, where the rods are oriented in a fixed direction. We formalise this model through a study of the structure of finite connected components in a high density supercritical regime and thereby establish the nematic behaviour as observed by Zwanzig.

This is work done jointly with Hideki Tanemura.

Anish Sarkar, Indian Statistical Institute, Delhi, India

Brownian Web in the Scaling Limit of Supercritical Oriented Percolation in Dimension 1 + 1

We prove that, after centering and diffusively rescaling space and time, the collection of rightmost infinite open paths in a supercritical oriented percolation configuration on the space-time lattice $\{(x, i) \in \mathbb{Z}^2 : x + i \text{ is even}\}$ converges in distribution to the Brownian web. This proves a conjecture of Wu and Zhang [WZ08]. Our key observation is that each rightmost infinite open path can be approximated by a percolation exploration cluster, and distict exploration clusters evolve independently before they intersect.

Volker Schmidt, Ulm University

Random Geometric Graphs Induced by Stationary Point Processes

For any locally finite set M in the Euclidean space of dimension at least 2, the minimal spanning forest MSF(M) is a generalization of the minimal spanning tree that was introduced originally by Aldous and Steele (1992). They conjectured that MSF(X) is almost surely connected if X is a homogeneous Poisson point process. This conjecture was proven in Alexander (1995) for dimension 2. However, it remains open for higher dimensions (and for non–Poisson point processes X).

In the present talk we introduce a family of approximations of the minimal spanning forest, see [1]. Using techniques of Blaszczyszyn and Yogeshwaran (2011) and Daley and Last (2005) we prove the a.s. connectivity of the constructed approximations for all stationary point processes X with finite range of dependence and absolutely continuous second factorial moment measure.

We also derive conditions for the a.s. finiteness of cells in the two-dimensional case and relate the results to stochastic modeling of telecommunication networks, which will be the subject of another talk presented during this workshop (by C. Gloaguen), see also [2] and [3].

[1] Hirsch, C., Neuhaeuser, D. and Schmidt, V. (2012) Connectivity of random geometric graphs related to minimal spanning forests. Preprint (submitted).

[2] Gloaguen, C., Voss, F. and Schmidt, V. (2011) Parametric distributions of connection lengths for the efficient analysis of fixed access networks. Annals of Telecommunications 66, 103-118.

[3] Neuhaeuser, D., Hirsch, C., Gloaguen, C. and Schmidt, V. (2012) On the distribution of typical shortest-path lengths in connected random geometric graphs. Queueing Systems (in print).

Evgeny Spodarev, Ulm University, Germany

Set reconstruction by Voronoi cells

Let A be a Borel set of finite volume in $\mathbb{R}^d$ and η be a homogeneous Poisson point process in $\mathbb{R}^d$. Assume that we observe η and the only information about A at our disposal is which points of η lie in A, i.e., we have the partition of the process η into $\eta \cap A$ and $\eta \setminus A$. We try to reconstruct the set A just by the information contained in these two point sets. For that we approximate A by the set A_η of all points in $\mathbb{R}^d$ which are closer to $\eta \cap A$ than to $\eta \setminus A$. In the talk we discuss the goodness of this approximation in terms of arbitrary moments of the volume of A_η and the symmetric difference $A \Delta A_\eta$. Namely, we investigate the asymptotics of these moments as the intensity of the underlying Poisson process goes to infinity for Borel sets A with finite volume and perimeter. In the proofs, the recent results of B. Galerne (2011) on the covariogram of sets with finite perimeter play a crucial role.

This is joint work with M. Reitzner and D. Zaporozhets.

Radu Stoica, Université Lille 1, France

Marked point processes : a tool for morphological and statistical characterisation of patterns in digital images and spatial data Les processus ponctuels marqués en analyse d'image

This talk presents the marked point processes as a methodological framework for morphological and statistical characterisation of the pattern "hidden" in digital images or more general in spatial data. This methodology is made of four tools : probabilistic modelling, Monte Carlo simulation, statistical inference and results validation. Examples from remote sensing, cosmology and epidemiology are also given. The speaker aims also to start a scientific debate with the public, about the main advantages and drawbacks of this approach.