PhD thesis proposal

Smoothed complexityof geometric structures and algorithms

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1 Context and motivation

Geometric problems are central in many areas of science and engineering. Computational geometry, the study of combinatorial and algorithmic problems in a geometric setting, and in particular triangulations have tremendous practical applications in areas such as computer graphics, computer vision and imaging, scientific visualization, geographic information systems, astronomy, computational biology... Traditionally, the complexity of computational geometry algorithms is studied in the worst case setting. This kind of analysis is often quite pessimistic compared to real life data. Due to the emergence of standardized software libraries, in particular the Computational Geometry Algorithms Library CGAL, developed in the framework of an Open Source Project, the so-far mostly theoretical results developed in computational geometry are being used and extended for practical use like never before. CGAL has been proposing efficient and robust packages computing Delaunay triangulations in the 2D and 3D Euclidean spaces for years.

2 Objectives

The goal of this research is to develop new probabilistic analysis for algorithms for convex hulls and triangulations. Such analysis should have consequences on the design of triangulation algorithms. See more details in the work program below.

3 Knowledge involved:

The candidate will have a strong background in probability (with geometric flavour if possible).

Ii will be good if the candidate have some knowledge in algorithmic aspects (e.g. sorting algorithms and their complexity).

Some knowledge of programming language is a plus (C++, templates, etc).

4 Collaborations and funding

The thesis will be co-advised by Pierre Calka (Univ. Rouen) and involve also some collaboration with INRIA-EPI Vegas. and will be funded by ANR project Présage `http://webloria.loria.fr/~goaoc/ANR-Presage/`.

The candidate will got a doctoral contract with INRIA (about 1600 euros/month) with the possibility of getting teaching duties (raising the salaty to 1875 euros).

5 Working program

In a preliminary work [1], we analyzed the expected complexity of the convex hull of small perturbations of "regular" samples of a sphere in Rd, obtaining tight estimates of the average number of vertices as a function of n, d, the radius of perturbation. In other words, we started analyzing the smoothed complexity of convex hulls, a task that proved surprisingly non-trivial. We propose to extend this work in the following directions.

Faces of all dimension. Our preliminary work [1] only address the number of extreme points under spherical noise. The number of faces of all dimensions can be attacked by the mean of higher degree probabilistic moments.

Different perturbation models. Our preliminary work [1] use spherical noise in all dimensions and cubical noise only in 2D. Generalizing to other shapes of noise such as cubical noise in higher dimension or Gaussian noise is an interesting problem. Cubical and Gaussian noise have the property to alter the different coordinates in an independent way, this property is crucial in results of Damerow and Sohler [3] that provides sub optimal results for this kind of noise.

Extremal configurations. In [1] we consider an initial configuration, which is a good sample of a sphere, that is perturbed by some noise; then the expected number of extreme points is computed. For some noise intensity, we know some initial configuration with points inside the sphere where the expected number of extreme points is worse. Thus a natural question is what is the worse initial situation for a given level of noise ?

Smoothed complexity of Delaunay triangulations The real interesting stuff is Delaunay (as applications show a significant gap between worst-case and practical complexity). Comes as a natural follow-up after convex hulls as there are many connections...

alpha-shapes in 3D. Definition. Natural first step as it interpolates between convex hull and Delaunay. To what extent do our technique for CH generalize to alpha-shapes? Quadratic configurations in 3D. Point sets can have quadratic DT in 3D but this requires them to essentially sit on a curve. Perturbing will therefore decrease the complexity as it will likely increase the «spread». Quantifiying the speed at which this decrease happens is an interesting question to understand whether noise explains why practical data never has quadratic-size DT. Do certain quadratic configurations have their complexity that decrease quicker than others?

Higher dimension, general bounds. Naturally, the big open problem is to identify all extremal configurations and analyze all their moments... Probably too much to ask, but we keep it as a long term goal.

References

[1] Dominique Attali, Olivier Devillers, and Xavier Goaoc. The effect of noise on the number of extreme points. Research Report 7134, INRIA, 2009. `http://hal.inria.fr/inria-00438409/`.

[2] P. Calka and T. Schreiber. Large deviation probabilities for the number of vertices of random polytopes in the ball. *Adv. in Appl. Probab.*, 38:47–58, 2006. `http://www.jstor.org/stable/20443427`.

[3] Valentina Damerow and Christian Sohler. Extreme points under random noise. In *Proc. 12th European Symposium on Algorithms*, pages 264–274, 2004. `http://www.uni-paderborn.de/fachbereich/AG/agmadh/PapersPostscript/VIO_extreme_points_esa.ps.gz`.

[4] The CGAL Project. *CGAL User and Reference Manual.* CGAL Editorial Board, 3.7 edition, 2010. `http://www.cgal.org/Manual/3.7/doc_html/cgal_manual/packages.htm%l`.